**Supplementary Results**

*Validation of single clutch ODEs against Monte Carlo simulations*

To test the validity of the single clutch ODEs (**Eqns. 1,2,4-7**), the ODE solution was compared to a fixed time step Monte Carlo model6 output for the case of one clutch and one motor. **Figure S1A** depicts the on and off cycles of the single clutch. The clutch is initially set in the unbound state and stochastically binds after some amount of time. Once bound, tension builds on the clutch, increasing its off-rate until it stochastically unbinds. This unbinding event signifies the end of one cycle and the start of another. After this unbinding, the clutch is free to bind again. **Figures S1B-F** present the average time evolution of the clutch binding and unbinding cycle over 1000 Monte Carlo runs and compares the Monte Carlo output to the ODE solution. In these figures, the ODE solution matches very well with the stochastic output. In **Figure S1B**, the probability that the clutch is bound rises at first, reaches a peak, and then settles over time to a steady state probability of being bound, given that the cycle has lasted to that time. Likewise, the substrate position (**Fig. S1C**), clutch force (**Fig. S1D**), and clutch off-rate (**Fig. S1F**) all rise to steady state values. Conversely, the actin retrograde flow rate (**Fig. S1E**) slows to a stalled steady state. This indicates that long cycles will result in no actin flow at the end of the cycle. Again, all of these steady state values are conditional on the cycle lasting until that time. However, cycles will likely fail before this steady state is reached since cycle times are ~5 s (**Fig. S1A**).

*Constraint on gamma distribution shape parameter (r)*

As described in the main text, use of the gamma distribution for the clutch forces imposes that  or . **Figure S2A** shows the minimum allowable value of *r* (), over the course of one Monte Carlo load-and-fail cycle. As the cycle progresses, the quantity  increases to about 2 before spiking at the time of failure, indicating a minimum overall value of *r* = 2. Other values of *r* were examined by plotting the quantity over the time-course of the cycle (**Fig. S2B**). Values of *r* for which the curve remains positive indicate the values for *r* which are allowed. Since the curve for *r* = 1 drops below 0, it is not an allowable value, but both *r* = 2 and *r* = 3 satisfy the constraint. 1000 Monte Carlo cycles were averaged to give **Figure S2C**, depicting smooth curves of the quantity over time.

By altering *r* on different stiffnesses, we can generate a curve of the minimum *r*-value as a function of stiffness. The value of *r* was stepped until the average  over 1000 Monte Carlo runs remained positive (within 1% of the maximum value) at all times. This value of *r* was taken as the minimum allowable *r* for that stiffness. **Figure S2D** presents this minimum value of *r* as a function of substrate stiffness. On soft substrates, the minimum *r*-value remains relatively constant at about 2 (the F-actin retrograde flow dependence on substrate stiffness is also shown for reference). At the highest stiffness sensitivity (i.e. when the change in actin flow is the greatest with respect to substrate stiffness) the minimum value of *r* begins to increase and continues increasing while in the frictional slippage regime. Since the model is insensitive to stiffness in the stiff frictional slippage regime, the change in the minimum *r*-value in this regime has little consequence.

The chosen value for *r* also affects the motor-clutch cycle time as represented by the fraction of bound clutches (**Fig. S2E**). Increasing *r* from 1 to 3 results in an increase of cycle time from 15 s to 73 s. **Figure S2F** compares the Monte Carlo output cycle time as a function of stiffness to the ODE solutions for different values of *r*. At low stiffness (< 1 pN/nm), *r* = 2 matches very well with the Monte Carlo output, but at intermediate stiffness (1-10 pN/nm) lower values of r are required to match the Monte Carlo cycle time. At high stiffness, the Monte Carlo time flattens, requiring higher values of *r* to match it. Fortunately, at these high stiffnesses, the motor-clutch behavior is insensitive to stiffness, so changing *r* has little effect on the overall behavior. Given the minimum r-values calculated for varying stiffness and the dependence of cycle time on *r*, values of *r* between 2 and 3 were used in this study.

The calculated minimum value of *r* does provide some insight into the system behavior. The minimum value of *r* increases with substrate stiffness, constraining the distribution to be closer to Gaussian in the high stiffness frictional slippage regime. On a stiff substrate, only a few clutches become engaged for a given cycle. In the limit of one engaged clutch, its force distribution is a delta function at the force on that particular clutch. As the clutch loads, this delta function moves to higher forces. The increase in *r* value on stiff substrates indicates this shift toward a narrow, approximately Gaussian distribution. On softer substrates, the distribution is allowed to be more skewed because of the lower minimum *r* value, and likely is more skewed because *r* ≤ 2 in order to match the Monte Carlo cycle time. On soft substrates where frictional slippage may occur during the loading cycle, *r* = 2 matches the Monte Carlo output indicating some skewness in the distribution with a mode at some non-zero force.

**Supplementary Figure S1: ODE and Monte Carlo behavior of a single clutch system.** The single clutch ODEs were validated again Monte Carlo simulations of motor-clutch systems with one clutch and one motor. A) The clutch stochastically binds and unbinds over time. Initially, the clutch is unbound and takes some time to bind. Force builds on the clutch increasing its off-rate until it unbinds defining the end of one cycle and the start of the next. B-F) Motor-clutch model outputs are presented for both the single clutch ODEs and the single clutch Monte Carlo simulation mean. The Monte Carlo output is shown as red dashed lines while the ODE solution is shown as solid blue lines. Monte Carlo output was averaged over 1000 motor-clutch cycles. All data presented is for the base parameter values on κsub = 0.1 pN/nm.

**Supplementary Figure S2: Analysis of the gamma distribution shape parameter.** A) As described in the model derivation, the use of the gamma distribution imposes the constraint that . Monte Carlo simulation outputs on κsub=0.1 pN/nm were used to calculate the minimum value of r over time. B) The same Monte Carlo output was used to calculate the quantity  for *r* = 1, *r* = 2, and *r* = 3. In order to adhere to the constraint, these curves must not drop below zero. For *r* = 1, the curve clearly goes negative, while *r* = 2 and *r* = 3 remain positive until the failure event at the end of the cycle. C) 1000 Monte Carlo cycles were averaged to obtain the average shape of the curves presented in B. D) At each substrate stiffness, the average curves of  were generated while stepping *r* up from zero until the curve remained above zero (within 1% of its maximum value). The first r resulting in this case was taken as the minimum allowable *r* for that stiffness. E) Increasing the value of *r* in the ODE solution increases the motor-clutch cycle time, here indicated by an abrupt drop in the fraction of clutches bound. F) Monte Carlo cycle time dependence on substrate stiffness is compared to ODE solution cycle times using different values of *r*. *r* = 2 matches the cycle time very well on stiffnesses below 1 pN/nm. In the range of 1-10 pN/nm, the Monte Carlo cycle time drops sharply and matches better to values of *r* between 0.3 and 2. Above 10 pN/nm, the Monte Carlo cycle time flattens, and requires increasingly high values of *r* to match it.

**Supplementary Table S1: Base parameter values**